

**I. PART 1. MULTIPLE CHOICE QUESTIONS (7,0 points)**

**Question 1:** Determine values of  $x$  in interval  $\left[-\pi; \frac{3\pi}{2}\right]$  such that function  $y = \tan x$  obtains a value of 0

- A.  $\left\{-\frac{\pi}{3}; \frac{\pi}{6}\right\}$       B.  $\{-\pi; 0; \pi\}$       C.  $\{-2\pi; 0; \pi\}$       D.  $\left\{-\frac{\pi}{4}; 0; \frac{\pi}{4}\right\}$

**Question 2:** How many different six digits numbers can be made from the digits 0, 1, 2, 3, 4 and 5 if each digit appears only once in the arrangement

- A. 300      B. 120      C. 600      D. 720

**Question 3:** Given regular tetrahedron  $S.ABC$  with every edge having length  $a$ . Let  $I$  be the midpoint of edge  $AB$  and  $M$  be a point moving on segment  $AI$ . Through point  $M$ , draw plane  $(\alpha)$  parallel to plane  $(SIC)$ . Let  $AM = x$ . The parameter of cross section created by the plane  $(\alpha)$  and tetrahedron  $ABCD$  is:

- A.  $3x(1+\sqrt{3})$       B.  $x(1+\sqrt{3})$       C.  $2x(1+\sqrt{3})$       D. Unabel to caculate

**Question 4:** From a box of six white and four black balls, take four random balls at one go. How many ways of doing such that four balls are the same colour?

- A. 210      B. 16      C. 15      D. 10

**Question 5:** In the  $Oxy$  coordinate plane, given line  $d$  whose equation is  $3x + 2y - 5 = 0$ . The image of the line  $d$  under a reflection across the  $Ox$ -axis has equation:

- A.  $3x + 2y + 5 = 0$       B.  $3x - 2y + 5 = 0$       C.  $3x + 2y - 5 = 0$       D.  $3x - 2y - 5 = 0$

**Question 6:** Let  $a$  be an integer. Knowing that equation  $x^2 - ax + 2a = 0$  has integer roots. Find the sum of the possible values of  $a$ .

- A. 8      B. 16      C. 17      D. 18

**Question 7:** Find a **false** statement in the following statements:

- A. If each of the two distinct planes is parallel to a third plane, then the two planes are parallel to each other.  
B. If each of the two distinct lines is parallel to a plane, then the two lines are parallel to each other.  
C. If two planes have a common point, then they have infinite numbers of other points in common.  
D. If a line intersects either of the parallel planes, then it intersects the other plane.

**Question 8:** Knowing that the coefficient of  $x^3$  in the expansion  $(1 - 2x)^n$  is 1760. Find  $n$ ?

- A.  $n = 11$       B.  $n = 9$       C.  $n = 12$       D.  $n = 10$

**Question 9:** Find a **true** statement in the following statements:

- A. If three non - collinear lines intersect in pairs, then they are concurrent.  
B. If three non - collinear lines intersect in pairs, then they form a triangle.  
C. If three non - collinear lines intersect in pairs, then they overlap.  
D. If three non - collinear lines intersect in pairs, then they are parallel to a plane.

**Question 10:** Points  $A(\sqrt{\pi}; a)$ ,  $B(\sqrt{\pi}; b)$  are distinct points on the graph of  $y^2 + x^4 = 2x^2y + 1$ . Find the value of  $M = |a - b|$ .

- A.  $M = 1$       B.  $M = 2$       C.  $M = \sqrt{1 + \pi}$       D.  $M = 1 + \sqrt{\pi}$

**Question 11:** Let  $a$ ,  $b$  and  $c$  be three distinct one-digit numbers. What is the maximum value of the sum of the roots of the equation  $(x - a)(x - b) + (x - b)(x - c) = 0$ ?

- A. 15      B. 15,5      C. 16      D. 16,5

**Question 12:** Given tetrahedron  $ABCD$ . Let  $M$  and  $N$  be the points of edges  $AB$  and  $AD$  with  $\frac{AM}{AB} = \frac{AN}{AD} = \frac{1}{3}$ . Let  $E$  be a point on edge  $CD$  with  $ED = 4EC$ . The cross section created by the plane  $(MNE)$  and tetrahedron  $ABCD$  is:

- A. Triangle  $MNE$ .
- B. Quadrilateral  $MNEF$  with any point  $F$  on edge  $BC$ .
- C. Parallelogram  $MNEF$  with point  $F$  on edge  $BC$  and  $EF \parallel BD$ .
- D. Trapezoid  $MNEF$  with point  $F$  on edge  $BC$  and  $EF \parallel BD$ .

**Question 13:** Find the coefficient of  $x^7$  in expansion of expression  $\left(x + \frac{2}{x^2}\right)^{10}$ .

- A. 100
- B. 10
- C. 20
- D. 200

**Question 14:** Roll a balanced and homogeneous dice. Suppose the  $b$ -spot appears. Consider equation  $x^2 + bx + 2 = 0$ . The probability such that the equation has an integer solutions is:

- A.  $\frac{1}{6}$
- B.  $\frac{1}{3}$
- C.  $\frac{1}{2}$
- D.  $\frac{2}{3}$

**Question 15:** Flip a balanced and homogeneous coin four times. The probability of tails appearing four times is

- A.  $\frac{1}{8}$
- B.  $\frac{1}{4}$
- C.  $\frac{1}{16}$
- D.  $\frac{3}{8}$

**Question 16:** In the  $Oxy$  coordinate plane, given a circle  $(C)$  with center  $I(-2; 1)$  and radius  $R = 3$ . The image of the circle  $(C)$  under a symmertry about center  $A(1; -3)$  has equation:

- A.  $(x+4)^2 + (y-7)^2 = 9$
- B.  $(x+4)^2 + (y-7)^2 = 3$
- C.  $(x-4)^2 + (y+7)^2 = 9$
- D.  $(x-4)^2 + (y+7)^2 = 3$

**Question 17:** Consider the set of all fractions  $\frac{x}{y}$ , where  $x$  and  $y$  are relatively prime positive numbers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

- A. 2
- B. 3
- C. 0
- D. 1

**Question 18:** The largest value of function  $f(x) = 2\sqrt{\cos x + 1} - 4$  is:

- A. -4
- B. -2
- C. 4
- D.  $2\sqrt{2} - 4$

**Question 19:** Four circles, no two of which are equal, have centered at  $A, B, C, D$ , and points  $P, Q$  lie on all four circles. The radius of circle  $A$  is  $\frac{5}{8}$  times the radius of circle  $B$ , and the radius of circle  $C$  is  $\frac{5}{8}$  times the radius of circle  $D$ . Furthermore,  $AB = CD = 39$  and  $PQ = 48$ . Let  $R$  be the midpoint of segment  $PQ$ . Find the value of  $AR + BR + CR + DR$ ?

- A. 192
- B. 196
- C. 184
- D. 188

**Question 20:** Given  $\tan a = 2$ . The value of expression  $Q = \frac{5\sin a + 2\cos a}{4\sin a - 3\cos a}$  is

- A.  $-\frac{5}{12}$
- B.  $-\frac{12}{5}$
- C.  $\frac{12}{5}$
- D.  $\frac{5}{12}$

**Question 21:** Given triangular prism  $ABC.A'B'C'$ . Let  $M, M'$  be the midpoints of edges  $BC$  and  $B'C'$ , respectively. Find the intersection line  $d$  of two planes  $(AB'M)$  and  $(ACM')$ .

- A.  $d$  passing through  $A, M$
- B.  $d$  passing through  $A, I$  with  $I$  is intersection point of  $B'M$  and  $CM'$
- C.  $d$  passing through  $A, M'$
- D.  $d$  passing through  $A$  and parallel to  $B'M$  and  $CM'$

**Question 22:** Equation  $\frac{\cos 4x}{\cos 2x} = \tan 2x$  has the number of solutions belonging to interval  $\left(0; \frac{\pi}{2}\right)$

- A. 4                                      B. 2                                      C. 5                                      D. 3

**Question 23:** From the expansion of expression  $(x^2 + 2x + 3)^{10}$  into a polynomial, calculate a sum of the coefficient of obtained polynomial.

- A.  $6^{10}$                                       B. 0                                      C.  $5^{10}$                                       D.  $4^{10}$

**Question 24:** The largest negative solution to equation  $2 \tan^2 x + 5 \tan x + 3 = 0$  is

- A.  $-\frac{\pi}{6}$                                       B.  $-\frac{\pi}{4}$                                       C.  $-\frac{5\pi}{6}$                                       D.  $-\frac{\pi}{3}$

**Question 25:** In a plane, how many rectangles can be formed from 5 parallel lines and 6 lines perpendicular to five parallel lines?

- A. 30                                      B. 60                                      C. 120                                      D. 150

**Question 26:** Which of the following expression to depend on  $x$

- A.  $\cos^2 x + \cos^2\left(\frac{2\pi}{3} + x\right) + \cos^2\left(\frac{2\pi}{3} - x\right)$                                       B.  $\frac{\cos\left(x + \frac{\pi}{3}\right) + \cos\left(x - \frac{\pi}{3}\right)}{\cot x - \cot \frac{x}{2}}$
- C.  $\cos\left(x - \frac{\pi}{3}\right) \cdot \cos\left(x + \frac{\pi}{4}\right) + \cos\left(x + \frac{\pi}{6}\right) \cdot \cos\left(x + \frac{3\pi}{4}\right)$                                       D.  $\frac{\cos x + 2 \cos 2x + \cos 3x}{\sin x + \sin 2x + \sin 3x} - \cot 2x$

**Question 27:** Given tetrahedron  $ABCD$ . Let  $M$  and  $N$  be the midpoints of edges  $AB$  and  $AC$ . Let  $E$  be a point on edge  $CD$  with  $ED = 4EC$ . The plane  $(MNE)$  intersects the edge  $BC$  at  $F$ . Compute the ratio  $\frac{BF}{BC}$ .

- A.  $\frac{BF}{BC} = \frac{5}{4}$                                       B.  $\frac{BF}{BC} = 4$                                       C.  $\frac{BF}{BC} = \frac{1}{5}$                                       D.  $\frac{BF}{BC} = \frac{4}{5}$

**Question 28:** Given a box of 20 balls numbered from 1 to 20. Take a random ball. Find the probability of the following event “a ball with a number which is not divisible by 6 is taken”.

- A.  $\frac{3}{20}$                                       B.  $\frac{1}{2}$                                       C.  $\frac{17}{20}$                                       D.  $\frac{4}{5}$

**Question 29:** The integers  $a, b$  and  $c$  are such that  $a + b + c = 6$  and  $ab + bc - ca - b^2 = 1$ . Determine all values of  $abc$ .

- A. 6 and 10                                      B. 6 and -6                                      C. 10                                      D. 6

**Question 30:** Given two parallel lines  $d_1$  and  $d_2$ . On  $d_1$  there are 10 distinctive points, on  $d_2$  there are  $n$  distinctive points. Knowing that there are 8550 trapezoid whose vertexes are given points. Then the value of  $n$  is

- A. 15                                      B. 10                                      C. 20                                      D. 25

**Question 31:** Given vector  $\vec{v} = (a; b)$  in the  $Oxy$  coordinate plane. For each  $M(x; y)$ , we get  $M'(x'; y')$  that is the image of point  $M$  under a translation through vector  $\vec{v}$ . Which is the coordinate expression of a translation  $T_{\vec{v}}$  in the following expression

- A.  $\begin{cases} x' = x + a \\ y' = y + b \end{cases}$                                       B.  $\begin{cases} x = a - x' \\ y = b - y' \end{cases}$                                       C.  $\begin{cases} x' = x + a \\ y' = y - b \end{cases}$                                       D.  $\begin{cases} x = x' + a \\ y = y' + b \end{cases}$

**Question 32:** How many integers  $x$  such that the point  $M(x; -x)$  is inside or on the circle of radius 10 centered at  $I(5; 5)$ .

- A. 11                                      B. 12                                      C. 13                                      D. 14

**Question 33:** Let  $f(x) = ax^2 - c$ , where  $a, c$  are real numbers. Suppose  $-4 \leq f(1) \leq -1$  and  $-1 \leq f(2) \leq 2$ . What is the sum of the maximum and minimum values of  $f(8)$ ?

- A. 120                                      B. 121                                      C. 122                                      D. 123

**Question 34:** On interval  $-\infty; 0$ , equation  $\sqrt{x^2 - 3x + 2} - \sqrt{x^2 - 4x + 3} = x - 1$  is equivalent to equation

- A.  $\sqrt{2-x} - \sqrt{3-x} = -\sqrt{1-x}$                                       B.  $\sqrt{2-x} - \sqrt{3-x} = 1$   
 C.  $\sqrt{x-2} - \sqrt{x-3} = -\sqrt{x-1}$                                       D.  $\sqrt{x-2} - \sqrt{x-3} = \sqrt{x-1}$

**Question 35:** The ratio of the length to the width of a rectangle is 4:3. If the rectangle has diagonal of length  $d$ , then the area may be expressed as  $kd^2$  for some constant  $k$ . Find  $k$ ?

- A.  $k = \frac{4}{7}$                                       B.  $k = \frac{3}{7}$                                       C.  $k = \frac{16}{25}$                                       D.  $k = \frac{12}{25}$

**II. PART II. PROBLEMS SOLVING (3,0 points)**

**Question 1. (1,0 point)**

For each  $k = 1, 2, 3, \dots, 2018$ , the equation  $x^2 - 2x - k^2 - k = 0$  has roots  $\alpha_1, \beta_1; \alpha_2, \beta_2; \dots; \alpha_{2018}, \beta_{2018}$ , respectively. Evaluate  $T = \frac{1}{\alpha_1} + \frac{1}{\beta_1} + \frac{1}{\alpha_2} + \frac{1}{\beta_2} + \dots + \frac{1}{\alpha_{2018}} + \frac{1}{\beta_{2018}}$ .

**Question 2. (1,0 point)**

How many integers  $x$  are there in  $0, 1, 2, \dots, 2018$  such that  $C_{2018}^x \geq C_{2018}^{999}$ ?

**Question 3. (1,0 point)**

Let  $a, b, c$  be the real numbers that satisfy the following conditions  $\begin{cases} 5 \geq a \geq b \geq c \geq 0 \\ a + b \leq 8 \\ a + b + c = 10. \end{cases}$

Prove that  $a^2 + b^2 + c^2 \leq 38$ .

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*Giám thị 1*.....

*Giám thị 2*.....