ENGLISH MATHEMATICS TRAINING COURSE

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ĐỀ THI HSG TOÁN TIẾNG ANH – SGD NAM ĐỊNH
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I. PART 1. MULTIPLE CHOICE QUESTIONS (7,0 points)

Question 1. Equation $2\sin 2x+\sqrt{2}\sin 4x=0$ has the number of solutions belonging to interval $\left[-\frac{\pi}{2};2\pi\right]$

A. 9 **B.** 11 **C.** 12 **D.** 10

Question 2. Determine values of x in interval $\left[-\pi\frac{3\pi}{2}\right]$ such that function $y=\cot\left(x+\frac{\pi}{4}\right)$ obtains a value of 0.

A. $\left\{-\frac{\pi}{4};\frac{\pi}{4}\right\}$ **B.** $\left\{\frac{3\pi}{4};\frac{\pi}{4};-\frac{5\pi}{4}\right\}$ **C.** $\left\{-\frac{\pi}{4};\frac{\pi}{4};\frac{3\pi}{4}\right\}$ **D.** $\left\{-\frac{3\pi}{4};\frac{\pi}{4};\frac{5\pi}{4}\right\}$

Question 3. Let a,b,c be real numbers such that $a^2+b^2+c^2=1$ and $a^3+b^3+c^3=1$. Determine the maxinmum values of $T=a+b+c$.

A. 2 **B.** 1 **C.** -1 **D.** 0

Question 4. There is a group of 10 people consisting 6 men and 4 women. There is a need for forming a delegation of 5 people. How many choices of forming a delegation of 3 men and 2 women are there?

A. 120 **B.** 10 **C.** 252 **D.** 5

Question 5. How many sequences which are bounded in the following sequences?

$$u_n=2n^2-1,\qquad u_n=\frac{1}{n(n+2)},\qquad u_n=\frac{1}{2n^2-1},\qquad u_n=\sin n+\cos n$$

A. 1 **B.** 2 **C.** 4 **D.** 3

Question 6. Given triangular prism $ABCA'B'C'$. Let I and J be the centroids of triangles ABC and $A'B'C'$ respectively. The cross section created by the plane (AIJ) and the given prism is

A. An isosceles triangle

B. A right triangle

C. A trapezoid

D. A parallelogram

Question 7. Knowing that $C_n^2C_n^{n-2}+2C_n^2C_n^3+C_n^3C_n^{n-3}=100$. Find n.

A. $n = 8$ **B.** $n = 10$ **C.** $n = 4$ **D.** $n = 14$

Question 8. The number of interger solutions x of the equation $(12x-1)(6x-1)(4x-1)(3x-1) = 330$ is

A. 2 **B.** 3 **C.** 1 **D.** 0

Question 9. Throw a balanced and homogeneous dice twice at random. Find the probability of the event that the total number of pips equals 8.

A. $\frac{1}{6}$ **B.** $\frac{5}{36}$ **C.** $\frac{1}{9}$ **D.** $\frac{1}{12}$

Question 10. Find the coefficient of x^8 in the expansion of expression $\left(2x^2 + \frac{1}{x}\right)^{10}$, $(x \neq 0)$.

A. 1344 **B.** 210 **C.** 13440 **D.** 3360

Question 11. From the expasion of expression $\left(x^3 - 2x - 3\right)^7$ into a polynomial, calculte a sum of the coefficient of obtained polynomial.

A. 16384 **B.** -2187 **C.** 2187 **D.** -16384

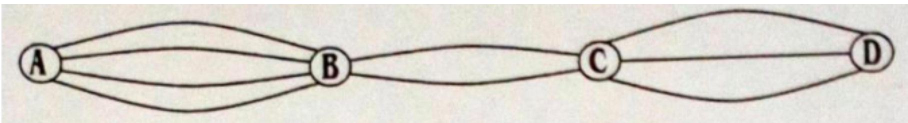
Question 12. Two boxes contain some balls. The first box contains 6 white and 4 black balls. The second box contains 4 white and 6 black balls. Take a random ball from each box. Calculated probability of the event that two balls from two boxes have different colours.

A. $11/25$ **B.** $12/25$ **C.** $13/25$ **D.** $14/25$

Question 13. The smallest value of function $f(x) = 4\sqrt{5 - \cos x} - 5$ is

A. 3 **B.** -1 **C.** 11 **D.** $4\sqrt{6} - 5$

Question 14. Cities A, B, C, D are linked by the roads as shown in Figure 1. How many ways of going from A to D via B and C once only are there?



A. 24 **B.** 18 **C.** 10 **D.** 9

Question 15. From digits $1, 2, 3, 4, 5$ and 6, how many natural numbers less than 100 can you make?

A. 42 **B.** 30 **C.** 36 **D.** 15

Question 16. Given a geometric sequence (u_n) with $u_1 = 3$ and common ratio $q = -2$. What ordinal of the term is number 192?

A. 8 **B.** 7 **C.** 5 **D.** 6

Question 17. Find the term not containing x in the expansion expression $\left(x^2 + \frac{1}{x}\right)^{12}$, $(x \neq 0)$

A. 924 **B.** 220 **C.** 495 **D.** 792

Question 18. Find a true statement in the following statements:

A. If a line cuts two given lines, then all three lines are coplanar.

B. If a line cuts two given intersecting lines, then all three lines are coplanar.

C. If three lines cut in pairs, they are conplanar.

D. If three lines cut in pairs and do not lie in the same plane, then they are concurrent.

Question 19. In triangle ABC with $AB = 3cm, BC = 4cm, CA = 5cm$. Three circles with respective centres A, B and C are pairwise tangent. A fourth circle is tangent to those three circles and contains all of them, as shown in the Figure 2. Calculate the radius, in cm, of the fourth circles.

A. $10cm$ **B.** $12cm$ **C.** $6cm$ **D.** $8cm$

Question 20. Given $\cot a = -2$. The value of expression $Q = \dfrac{\sin a - 3\cos a}{5\sin a + \cos a}$ is

A. $\frac{-5}{7}$ **B.** $\frac{-7}{3}$ **C.** $\frac{7}{3}$ **D.** $\frac{5}{7}$

Question 21. The first floor surface of a house is $0,5m$ higher than the yard surface. The staircase to the second floor consists of 21 stairs, each of which is $18cm$ in height. Calculate the height of the second floor as compared with the yard surface.

A. $4,28m$ **B.** $4,10m$ **C.** $3,785m$ **D.** $3,83m$

Question 22. Equation $\cos x = \sin x$ has the number of solutions belonging to interval $[-\pi; \pi]$

A. 5 **B.** 2 **C.** 1 **D.** 4

Question 23. Find the domain of function $y = \tan\left(x - \frac{\pi}{3}\right)$.

A. $\mathcal{D} = \mathbb{R}\backslash\left\{\frac{\pi}{6} + k\pi, k \in \mathbb{Z}\right\}$ **C.** $\mathcal{D} = \mathbb{R}\backslash\left\{\frac{5\pi}{6} + k\pi, k \in \mathbb{Z}\right\}$

B. $\mathcal{D} = \mathbb{R}\backslash\left\{\frac{\pi}{3} + k\pi, k \in \mathbb{Z}\right\}$ **D.** $\mathcal{D} = \mathbb{R}\backslash\left\{\frac{\pi}{2} + k\pi, k \in \mathbb{Z}\right\}$

Question 24. The largest negative solution to equation $\sin^2 x + \dfrac{\sqrt{3}-1}{4}\sin 2x - \dfrac{\sqrt{3}-1}{2}\cos^2 x = \dfrac{1}{2}$ is

A. $-\frac{\pi}{6}$ **B.** $-\frac{\pi}{4}$ **C.** $-\frac{2\pi}{3}$ **D.** $-\frac{\pi}{3}$

Question 25. In the Oxy coordinate plane, given point $M(x; y)$. We get $M'(x'; y')$ that is the image of point M under a symmetry about center $O(0; 0)$. Which is the coordinate expression of origin symmetry in the following expression?

A. $\begin{cases} x = x' \\ y = y' \end{cases}$ **B.** $\begin{cases} x = -x' \\ y = -y' \end{cases}$ **C.** $\begin{cases} x = -x' \\ y = y' \end{cases}$ **D.** $\begin{cases} x = x' \\ y = -y' \end{cases}$

Question 26. In the Oxy coordinate plane, given a circle $I(1; -1)$ and radius $R = 2$. The image of the circle (C) under a homothery with center $A(-2; 0)$ and ratio $k = -1$ has equation

A. $(x+5)^2+(y-1)^2=4$

B. $(x-5)^2+(y-1)^2=2$

C. $(x+5)^2+(y+1)^2=4$

D. $(x+5)^2+(y+1)^2=2$

Question 27. Throw a balanced and homogence twice at random. Find the probability of the event that there is at least one apearance of the five-spot.

A. $\frac{1}{3}$ **B.** $\frac{1}{9}$ **C.** $\frac{11}{36}$ **D.** $\frac{1}{6}$

Question 28. Find the *true* statement in the following statements:

A. If two planes (P) and (Q) are parallel, then every line in plane (P) is parallel to plane (Q).

B. If two planes (P) and (Q) are parallel, then every line in plane (P) is parallel to every line in plane (Q).

C. If two parallel lines in two distinct planes (P) and (Q) respectively, then planes (P) and (Q) are parallel.

D. Through a point outside a given plane, we can draw one and only one line parallel to the given plane.

Question 29. Find the *true* statement in the following statements:

A. $C_{n-1}^{k-1}+C_{n}^{k-1}=C_{n+1}^{k}, (0\leqslant k\leqslant n)$

B. $C_{n-1}^{k-1}+C_{n-1}^{k}=C_{n}^{k}, (0\leqslant k\leqslant n)$

C. $C_{n-1}^{k-1}+C_{n-1}^{k+1}=C_{n}^{k}, (0\leqslant k\leqslant n)$

D. $C_{n-1}^{k-1}+C_{n-1}^{k-2}=C_{n}^{k}, (0\leqslant k\leqslant n)$

Question 30. How many real numbers $a\in[1;9]$ such that the corresponding number $a-\frac{1}{a}$ is an interger?

A. 1 **B.** 0 **C.** 8 **D.** 9

Question 31. A bag has one white ball, one yellow ball and one red ball. A ball is drawn and then put back. Another ball is drawn next. Calculated probability of the event that there is at least one red.

A. $\frac{1}{3}$ **B.** $\frac{5}{9}$ **C.** $\frac{2}{9}$ **D.** $\frac{4}{9}$

Question 32. Given regular tetrahedron $ABCD$ with every edge having length a. Let I, J be the midpoints of edges AC, BC respectively and M be a point on edge BD with $MB=2MD$. The parameter of cross section created by the plane (MIJ) and tetraheron $ABCD$ is

A. $\frac{a(5+2\sqrt{13})}{6}$ **B.** $\frac{a(5+2\sqrt{6})}{3}$ **C.** $\frac{a(5+\sqrt{13})}{6}$ **D.** $\frac{a(5+2\sqrt{37})}{6}$

Question 33. Given pyramid $S.ABCD$ whose base is a rhombus $ABCD$. Let M, N, P be the midpoints of edges SA, SB, BC respectively. Knowing that $AB=a, SA=SB=a, SC=SD=a\sqrt{3}$. The area of cross section created by the plane (MNP) and pyramid $S.ABCD$ is

A. $\frac{3a^2\sqrt{19}}{16}$ **B.** $\frac{3a^2\sqrt{11}}{16}$ **C.** $\frac{3a^2\sqrt{2}}{8}$ **D.** $\frac{3a^2\sqrt{5}}{16}$

Question 34. In the Oxy coordinate plane, given point $A(2;0)$. Find B that is the image of point A under a rotation of -90° about center $O(0;0)$.

A. $B(0; 2)$ **B.** $B(2; 2)$ **C.** $B(-2; 0)$ **D.** $B(0; -2)$

Question 35. Given tetraheron $ABCD$. Let M, N, Q be points on edges AB, AD, BC respectively with $MA = MB; NA = 2ND; QB = 4QC$. The plane (MNQ) intersects the edge CD at P. Compute the ratio $\frac{DP}{DC}$.

A. $\frac{DP}{DC} = \frac{2}{3}$ **B.** $\frac{DP}{DC} = 2$ **C.** $\frac{DP}{DC} = \frac{1}{2}$ **D.** $\frac{DP}{DC} = \frac{3}{2}$

II. PART 2. PROBLEMS SOLVING (3,0 points)

Question 1. Let α be the larger root of equation $(2019x)^2 - 2018 \cdot 2020x - 1 = 0$ and β be the smaller root of equation $x^2 + 2018x - 2019 = 0$. Determine the value of $M = \alpha - \beta$.

Question 2. How many pairs (a, b) of positive integer are there such that $a \leqslant b$ and $2\left(\sqrt{\frac{15}{a}} + \sqrt{\frac{15}{b}}\right)$ is an integer?

Question 3. Let $a, b, c \in [1; 3]$ and satisfy the following conditions

$$\begin{cases} \max\{a, b, c\} \geqslant 2 \\ a + b + c = 5. \end{cases}$$

Find the smallest possible value of $T = a^2 + b^2 + c^2$.