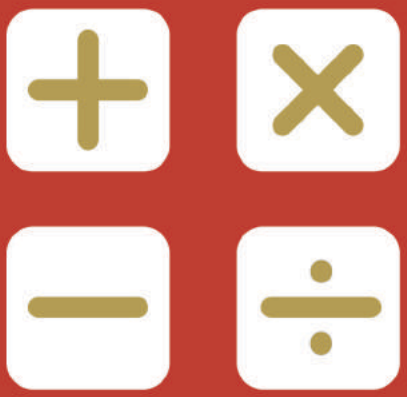
TCIMO | TERRY CHEW INSTITUTE OF MATHEMATICAL OLYMPIADS

PAPER F



SEAMO

Southeast Asian Mathematical Olympiads

SAMPLE

DO NOT OPEN THIS BOOKLET UNTIL INSTRUCTED.

STUDENT'S NAME:

Read the instructions on the **ANSWER SHEET** and fill in your **NAME, SCHOOL** and **OTHER INFORMATION**.
Use a 2B or B pencil.
Do **NOT** use a pen
Rub out any mistakes completely.

You **MUST** record your answers on the **ANSWER SHEET**.

SENIOR

Mark only **ONE** answer for each question.
Marks are **NOT** deducted for incorrect answers.

SECTION A
Use the information provided to choose the **BEST** answer from the five possible options.
On your **ANSWER SHEET** fill in the oval that matches your answer.

SECTION B
On your **ANSWER SHEET** fill in your answer within the box provided.

You are **NOT** allowed to use a calculator.

1. Evaluate.

$$\frac{1 + 3 + 5 + \cdots\cdots + 2013 + 2015}{2 + 4 + 6 + \cdots\cdots + 2014 + 2016}$$

(A) $\frac{2016}{2018}$

(B) $\frac{1004}{1005}$

(C) $\frac{1008}{1009}$

(D) $\frac{1002}{1003}$

(E) $\frac{2015}{2017}$

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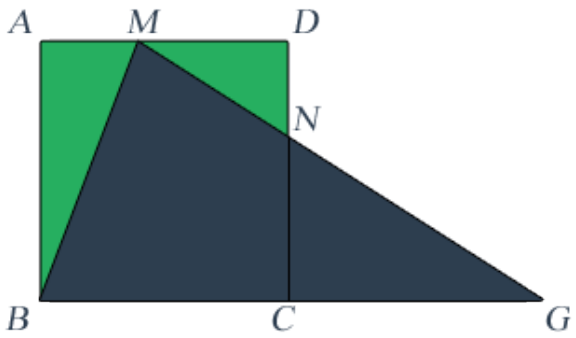
2. In the diagram shown below, $ABCD$ is a square with $NC = 2DN$. M is a point on AD, such that $\angle NMB = \angle MBC$. Find $\tan\angle ABM$.



(A) $\frac{2}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{4}$

(E) $\frac{1}{5}$

3. There is a real root, $0^\circ \ll \alpha \ll 180^\circ$, for $(3\sin a)x^2 - (4\cos a)x + 2 = 0$, find the range of $\sin a$.

(A) $0 \ll \sin a \ll \frac{1}{2}$

(B) $0 \ll \sin a \ll \frac{\sqrt{3}}{2}$

(C) $0 \ll \sin a \ll 1$

(D) $\frac{1}{2} \ll \sin a \ll \frac{\sqrt{3}}{2}$

(E) $\frac{1}{2} \ll \sin a \ll 1$

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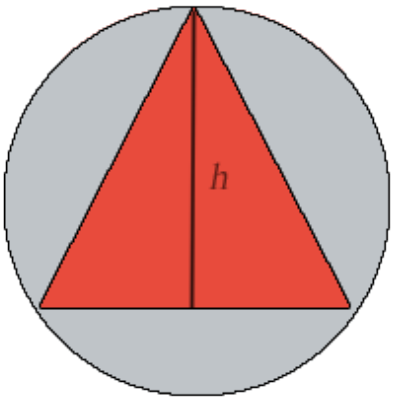
4. An equilateral triangle in inscribed in a circle. When the radius is r, the area of the triangle is greatest when its height is



(A) $1.2r$

(B) $1.3r$

(C) $1.4r$

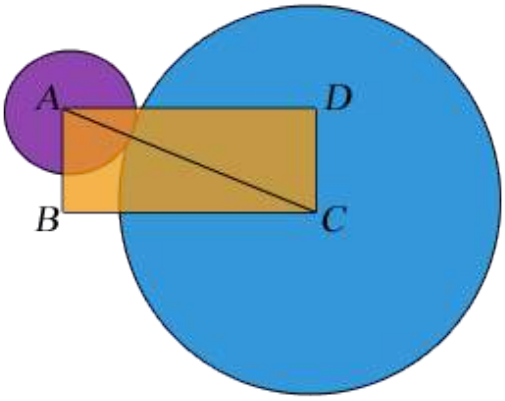
(D) $1.5r$

(E) $1.6r$

QUESTION 5 IS FREE RESPONSE

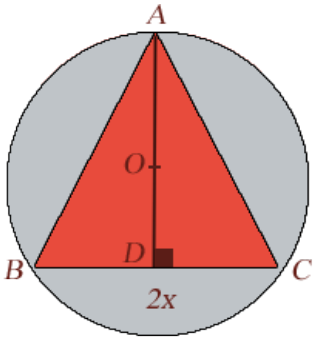
Write your answer in the boxes provided on the ANSWER SHEET and fill in the ovals that match your answer.

5. In the diagram below, r is the radius of the small circle with center A. $ABCD$ is a rectangle with $AB = 5\ cm$ and $BC = 12\ cm$. Find the range of values of r.



END OF PAPER

QUESTION	ANSWER	SOLUTION	TOPIC	DIFFICULTY
1	C	$\frac{1+3+5+\cdots+2015}{2+4+6+\cdots+2016}$ ➔ $\frac{\frac{(1+2015)\text{ x }1008}{2}}{\frac{(2+2016)\text{x }1008}{2}} = \frac{2016}{2018}$ ➔ $\frac{1008}{1009}$	Even and Odd Numbers	**Easy**
2	E	Let $AB \equiv 3, \because NC \equiv 2DN$ $\therefore DN \equiv 1, NC \equiv 2$ $\because MD \,//\, CG$ $\therefore \Delta MDN \sim \Delta GCN$ $\frac{MP}{CG} = \frac{MN}{NG} = \frac{DN}{NC} = \frac{1}{2}$ Let $MD = k, CG = 2k,$ $MN = \sqrt{1+k^2}$ $NG = 2\sqrt{1+k^2}$ $\therefore MG = 3\sqrt{1+k^2}$ $\because \angle NMB = \angle MBC,$ $\therefore GM = GB,$ $3\sqrt{1+k^2} = 2k+3 \rightarrow k = \frac{12}{5}$ $\therefore \tan\angle ABM = \frac{AM}{AB}$ $= \frac{3-\frac{12}{5}}{3}$ $= \frac{1}{5}$	Trigo-nometry	**Medium**

3	A	For $\sin a \neq 0$ Then $16\cos^2 a - 4(3\sin a)$ x $2 \geq 0$, $16\,(1-\sin^2 a) - 24\ \sin^2 a\ \geq 0$, $2\ \sin^2 a + 3\sin a - 2 \leq 0$ $\therefore (\sin a + 2)(2\sin a - 1) \leq 0$ $\sin a\ \leq \frac{1}{2}$	Trigo-nometric Identities	**Medium**
4	D	Let the base be $2x$  $h = AO + OD$ $= r + \sqrt{(r^2 - x^2}$ ➔ $x^2 = h(2r - h)$ Area of $\Delta ABC = \frac{1}{2}\,.\,2x\,.\,h$ $= \sqrt{2\,.h - h^2}\,.h$ $= \sqrt{2rh^3 - h^4}$ Let area of ΔABC be y $\frac{dy}{dh} = \frac{1}{2}\,.(2rh^2 - h^4)^{-\frac{1}{2}}(6rh^3 - 4h^3)$ $= \frac{h^2(3-2h)}{\sqrt{(2r-h)h^3}}$ Thus, when $\frac{dy}{dh} = 0,\ r = \frac{3}{2}r$	Pythago-rean Theorem	**Medium/Hard**

5	$1 < r < 8$	Since $ABCD$ is a rectangle, $AB = 5$ and $BC = 12$ $AC = \sqrt{5^2 + 12^2}$ $= 13$ $5 < R < 13$ $\because R + r = 13$ $R = 13 - r$ $\therefore 1 < r < 8$	Pythago-rean Theorem	**Medium/Hard**

Level of difficulty refers to the expected level of difficulty for the question.	
Easy	more than 75% of candidates will choose the correct option
Medium	about 50–75% of candidates will choose the correct option
Medium/Hard	about 25–50% of candidates will choose the correct option
Hard	less than 25% of candidates will choose the correct option